Binary Decision Diagrams in Reliability Theory

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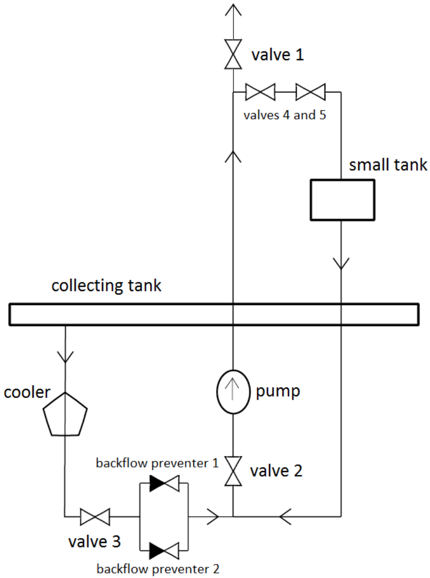
Outline

1. Motivation
   - small example
   - stating the objectives
2. Introduction of BDDs
3. Evaluating effectivness of BDDs

Temelín Nuclear Power Station – small example



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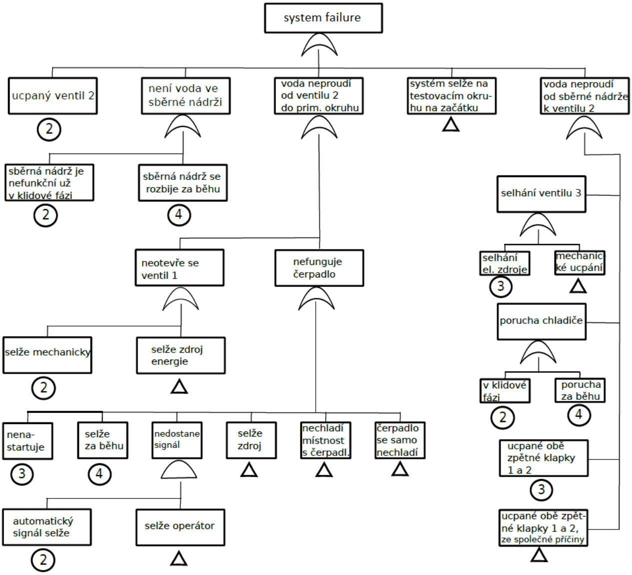
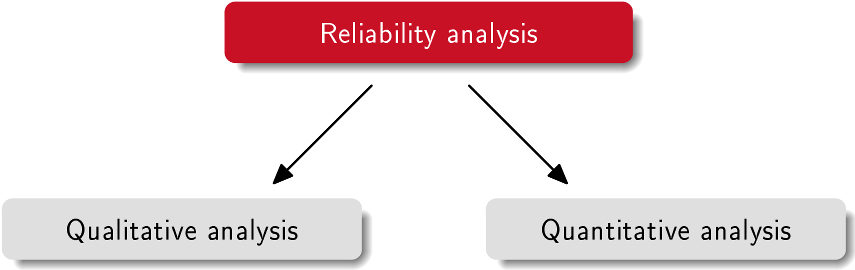


Two types of analyses



Qualitative analysis

- we just know whether a component (or a whole system) works or does not work
- it makes sense to consider components as boolean variables

$$x_i = \begin{cases} 0, & \text{if the component is working,} \\ 1, & \text{if the component is broken} \end{cases}$$

Then the functionality of the system consisting of (basic) components $x_1, \dots, x_n$ is given by a boolean function, so-called structure function.

Structure function S

$$S(x_1, \dots, x_n) = \begin{cases} 0, & \text{if the system is working,} \\ 1, & \text{if the system is broken} \end{cases}$$

Minimal solutions

Solution

Combination of component's failures that can cause failure of the whole system.

Minimal solution

The smallest combination of component's failures that can cause failure of the whole system.

Minimal solution set

Set of all minimal solutions.

Shannon's decomposition

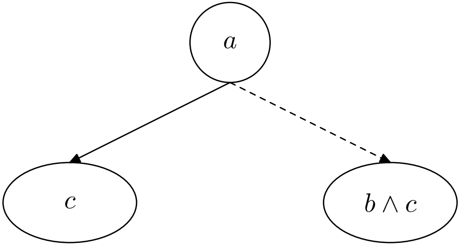
Theorem (Shannon's decomposition)

For boolean function $f = f(x_1, \dots, x_n)$ and for $i \in \{1, \dots, n\}$ the following equality holds $f = (x_i \wedge f_{\{x_i=1\}}) \vee (\neg x_i \wedge f_{\{x_i=0\}})$.

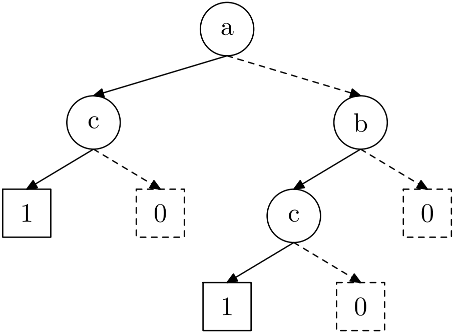
- For example:

$$(a \vee b) \wedge c = (a \wedge c) \vee (\neg a \wedge b \wedge c)$$

- Graphically represented:

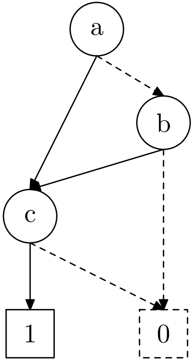


- Let's choose an ordering on variables and conduct Shannon's decomposition in this order.
- For our function $f(a, b, c) = (a \vee b) \wedge c$ we choose naturally $a < b < c$.
- The result is a *BDD*:



Reduced binary decision diagram (RBDD)

- After removing redundancies, the same function $f(a, b, c) = (a \vee b) \wedge c$ is more conveniently represented by:



Reduced binary decision diagram (RBDD)

- For given boolean function, there is a unique RBDD representing it (when variable ordering is fixed)*.
- Therefore, tautology test (well-known NP-complete problem) becomes trivial after obtaining RBDD representation of a function.

$\Rightarrow$ Construction of RBDD must be a NP-hard problem.

* BRYANT, Randal E. (1986) Graph-based algorithms for boolean function manipulation. *Computers, IEEE Transactions on*, 100.8: 677-691.

Utilization of RBDDs

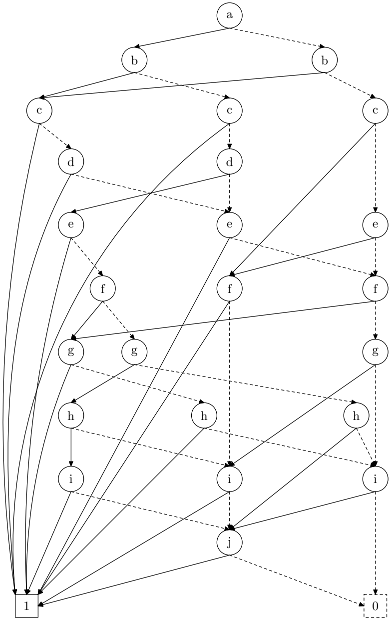
(1) RBDD is a suitable representation of boolean function in computer memory.

(2) RBDDs have a lot of *nice* properties.

(3) RBDDs bring new options for reliability analysis.

- We can find minimal solution set of a RBDD more effectively.
- Afterwards, probability of system failure is trivial to compute.

Temelín Nuclear Power Station – small example



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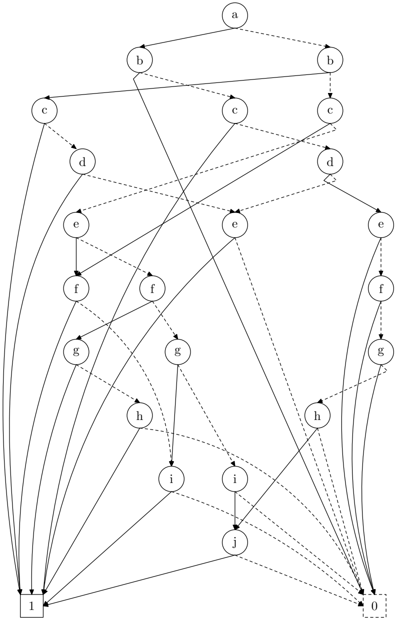

Minimal solutions

Minimal solution

The smallest combination of component's failures that can cause failure of the whole system.

- Usually the main purpose of reliability analysis.
- Provide useful insight into complex or large systems.
- Make subsequent analyses much easier.

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(− ? +) an efficient implementation is complex and requires careful memory representation

Thank you

Examples used with a kind permission of Department of Risk Assessment, Temelín Nuclear Power Station